CHAPTER 4 -- KINEMATICS

4.1)

a.) The *total* distance traversed (versus the net displacement) divided by the elapsed time. That scalar is:

$$\begin{aligned} s &= \text{dist/time} \\ &= (440\ \text{m})/(49\ \text{sec}) \\ &= 8.98\ \text{m/s}. \end{aligned}$$

b.) The magnitude of the *average* <u>*velocity*</u> is the *net displacement* divided by the elapsed time. That is:

$$\begin{aligned} \mathbf{v} &= \text{(net disp)/time} \\ &= (0\ \text{m})/(49\ \text{sec}) \\ &= 0\ \text{m/s}. \end{aligned}$$

Making sense of this: The woman finished where she started, so her net displacement is zero. The average velocity tells us the constant velocity she would have to travel to effect that displacement in 49 seconds. That velocity is zero!

c.) You know nothing about her instantaneous velocity at any point in the motion--not even at the beginning (for all you know, she may have had a running start).

4.2) The only options we have are:

a.) The curve could be an *acceleration versus time* graph. For a given *time interval*, the area under such a curve yields the *velocity change* over the interval. As we have *not* been told about *time intervals* or *velocity changes*, we don't have enough information to say *yea* or *nay*.

b.) The curve could be a *velocity versus time* curve. If that be so, the velocity magnitude at $t = 1$ *second* should be $v = -1$ *m/s*. Unfortunately, the graph reads +1 at $t = 1$ *second*. The graph is not a *velocity versus time* graph.

c.) The slope of the tangent to a *distance versus time* graph at any given point equals the *instantaneous velocity* at that point in time. Taking the curve's slope at $t = 1$ *second* we get -1. The curve must, therefore, be a *position versus time* graph.

4.3)

a.) During a given time interval, the *net displacement* of a body is equal to the area under the *velocity versus time* curve. The *area under the curve* between $t = .5$ *seconds* and $t = 3$ *seconds* has two parts: one above the axis and one below the axis.

Noting that one square on the graph is equal to 1/8 meter, the *area above the axis* is eyeballed at approximately (+2.3 squares)(1/8 meter/square), or +.29 meters. The *area below the axis* is eyeballed at approximately (-22 squ)(1/8 meter/squ), or -2.75 meters.

The net distance traveled is approximately (.29 m) + (-2.75 m) = -2.46 meters. That is, the ant travels 2.46 meters to the left of its starting point.

Note 1: This number is *not* the ant's final position-coordinate. It is only the net distance the ant traveled *from its original position* during the time interval.

Note 2: Written out fully, $\Delta \mathbf{x} = -2.46\mathbf{i}$ *meters*.

b.) *Average velocity* is defined as the *net displacement per unit time* over a time interval. In *Part a*, we determined the ant's *net displacement* between the .5 second and 3 second mark as -2.46 meters. As the *time interval* is 2.5 seconds:

$$\begin{aligned} v_{avg} &= (\text{net disp})/(\text{time interval}) \\ &= (-2.46\mathbf{i} \text{ meters})/(2.5 \text{ seconds}) \\ &= -.98\mathbf{i} \text{ m/s}. \end{aligned}$$

c.) Taking the information directly off the graph, the ant's velocity:

i.) At $t = .5$ *seconds* is approximately 1.3**i** m/s.

ii.) At $t = 3$ *seconds* is approximately -2.1**i** m/s.

d.) At a given point in time, the acceleration (i.e., the change of velocity with time) is the *slope of the tangent to the velocity curve*. Eyeballing it (watch the graph scaling), the slope of the tangent at:

i.) $t = .5$ *seconds* is approximately (-2)/(.5), or $\mathbf{a} = -4\mathbf{i}\ m/s^2$.

ii.) $t = 3$ *seconds* is approximately (1)/(2.25), or $\mathbf{a} = +.44\mathbf{i}\ m/s^2$.

e.) When the velocity is positive, the ant is moving in the *+x direction* (the direction of motion is the direction of the instantaneous velocity). This occurs between *t = .3 seconds* and *t = 1 second*.

f.) When the velocity is zero, the ant is standing still. This occurs at *t = 1 second*.

g.) The acceleration is zero when the slope of the tangent to the curve is zero (that is, when the velocity is not changing). This occurs at times *t = 2.9 seconds* and *t = 3.6 seconds*.

h.) When the *slope* of the velocity graph *changes*, we have what are called *inflection points*. To determine one, we need to observe two things:

i.) When the acceleration is *not changing*, the acceleration is constant.

ii.) A constant acceleration generates a *velocity function* that *changes linearly* (i.e., the velocity changes at a constant rate).

The interval over which the velocity seems to be *changing* linearly is between *t = .3 seconds* and *t = .5 seconds*, at *3.25 seconds*, and maybe between *t = 4 seconds* and *t = 4.3 seconds* (though this latter suggestion is debatable).

4.4) The velocity function is $\mathbf{v}(t) = (3e^{-1.5kt}\mathbf{i} - 4k_1t\mathbf{j})$ m/s. Noting that the constant $k = 1\ sec^{-1}$ (the only way the exponent in e^{-kt} can be unitless is if *k's* units are *1/seconds*) and $k_1 = 1\ m/s^2$:

a.) Omitting the *k* terms (their magnitudes are "1"--they are present only for the sake of units), we can plug *t = 2 seconds* into the *velocity function* yielding:

$$\begin{aligned}\mathbf{v}(t) &= (3e^{-1.5t}\mathbf{i} - 4t\mathbf{j}) \text{ m/s.}\\ &= [3e^{-(1.5)(2)}\mathbf{i} - 4(2)\mathbf{j}] \text{ m/s}\\ &= [.15\mathbf{i} - 8\mathbf{j}] \text{ m/s.}\end{aligned}$$

b.) *Acceleration* is the *time derivative* of the *velocity function*, or:

$$\begin{aligned}\mathbf{a}(t) &= d\mathbf{v}(t)/dt\\ &= d(3e^{-1.5t}\mathbf{i} - 4t\mathbf{j}) / dt\\ &= [-4.5e^{-1.5t}\mathbf{i} - 4\mathbf{j}] \text{ m/s}^2.\end{aligned}$$

c.) Plugging *t = 2 seconds* into the acceleration function yields:

$$\mathbf{a} = [-4.5e^{-1.5(2)}\mathbf{i} - 4\mathbf{j}]\ \text{m/s}^2.$$
$$= [-.22\mathbf{i} - 4\mathbf{j}]\ \text{m/s}^2.$$

d.) Using the *kinematic equations* requires a *constant acceleration*. In this case, there are *two* accelerations to consider--one in the *x direction* and one in the *y direction* (they are independent of one another).

i.) The *x component* of acceleration is *time dependent*. That is, it is not a constant. As such, the *kinematic equations* will <u>not</u> be valid for the body's *x-type motion*.

ii.) The *y component* of the acceleration is constant ($a_y = 4\ m/s^2$). Kinematics <u>will</u> work on problems associated with the body's *y-type motion*.

e.) The body's *position function* ***r*** is determined by integrating the *velocity function* ***v****(t)* over an indefinite time interval (i.e., by solving $\int(\mathbf{v})dt$ without evaluating between times t_1 and t_2). Executing that operation, we get:

$$\begin{aligned}\mathbf{r}(t) &= \int \mathbf{v}(t)dt \\ &= \int \left[(3e^{-1.5t}\mathbf{i} - 4t\mathbf{j}\right]dt \\ &= \int \left[(3e^{-1.5t}\mathbf{i}\right]dt + \int \left[(-4t\mathbf{j}\right]dt \\ &= \left(\left[-2e^{-1.5t}\mathbf{i}\right] + \left[-2t^2\mathbf{j}\right]\right) \text{ meters.}\end{aligned}$$

Notice there is no evaluation done here. All the problem asked for was the function that defines the position ***r****(t)* for the body. Note also that the derivative of ***r****(t)* yields ***v****(t)*.

f.) To find the *displacement*, we must evaluate the *position function* at *t = 2 seconds* and *t = 3.5 seconds*, then subtract the two to get the net *change* of position (that is what the *net displacement* is). Doing so yields:

$$\begin{aligned}\Delta\mathbf{r} &= \left[\left[-2e^{-1.5t}\mathbf{i}\right] + \left[-2t^2\mathbf{j}\right]\right]_{t=2\ \text{sec}}^{3.5} \\ &= \left(\left[-2e^{-1.5(3.5)}\mathbf{i}\right] + \left[-2(3.5)^2\mathbf{j}\right]\right) - \left(\left[-2e^{-1.5(2)}\mathbf{i}\right] + \left[-2(2)^2\mathbf{j}\right]\right) \\ &= (-.01\mathbf{i} - 24.5\mathbf{j}) - (-.10\mathbf{i} - 8\mathbf{j}) \\ &= [.09\mathbf{i} - 16.5\mathbf{j}]\ \text{m.}\end{aligned}$$

g.) The change of the body's position between $t = 2$ *seconds,* and $t = 3.5$ *seconds* is $\Delta\mathbf{r} = .09\mathbf{i} - 16.5\mathbf{j}$. This change of position can also be written as $\Delta\mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$. As the *x-type motion* and the *y-type motion* are independent of one another, we can separate the two and write:

<u>for **i** motion:</u>

$$\begin{aligned}(.09\text{ m}) &= (x_2 - x_1)\\ &= x_2 - (-.1\text{ m})\\ \Rightarrow \quad x_2 &= -.01\text{ meters.}\end{aligned}$$

<u>for **j** motion:</u>

$$\begin{aligned}(-16.5\text{ m}) &= (y_2 - y_1)\\ &= y_2 - (-8\text{ m})\\ \Rightarrow \quad y_2 &= -24.5\text{ meters.}\end{aligned}$$

Coordinates at $t = 3.5$ *seconds* will be $x_2 = -.01$ *meters,* $y_2 = -24.5$ *meters.*

h.) The *jerk* is the *time rate of change of the acceleration.* That is:

$$\begin{aligned}\text{jerk} &= d\mathbf{a}/dt\\ &= d[-4.5e^{-1.5t}\mathbf{i} - 4\mathbf{j}] / dt\\ &= [6.75e^{-1.5t}\mathbf{i}]\text{ m/s}^3.\end{aligned}$$

i.) Exponents must be unitless, so k must have the units of *seconds*$^{-1}$.

4.5) We know the bats' initial vertical velocity $v_1 = 0$, their initial height $h_1 = 100$ *meters*, their pull-out height $h_2 = 3$ *meters*, and their acceleration a is <u>minus</u> the magnitude of the acceleration of gravity, or $a = -g = -9.8\text{ m/s}^2$:

a.) We want v_2. Scanning the kinematic equations (shown below),

$$(x_2 - x_1) = v_1\Delta t + (1/2)a(\Delta t)^2$$

$$(x_2 - x_1) = v_{avg}\Delta t \quad \text{or} \quad v_{avg} = (x_2 - x_1)/\Delta t$$

$$v_{avg} = (v_2 + v_1)/2$$

$$a = (v_2 - v_1)/\Delta t \quad \text{or} \quad v_2 = v_1 + a\Delta t$$

$$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1),$$

we decide to use:

$$(v_2)^2 = (v_1)^2 + 2a(x_2 - x_1)$$
$$\Rightarrow \quad v_2 = [(0)^2 + 2(-9.8 \text{ m/s}^2)[(3 \text{ m}) - (100 \text{ m})]]^{1/2}$$
$$= 43.6 \text{ m/s}.$$

Note: This equation yields the *magnitude* of the velocity only (the quantity v_2 is *squared* in the equation). If we use this value in subsequent problems, we must make it negative (the body is moving downward in the ***-j** direction*).

$$\Rightarrow \quad \mathbf{v}_2 = -43.6\mathbf{j} \text{ m/s}.$$

b.) We know that at 3 meters above the ground, $y_2 = 3$ *meters* and $v_2 = -43.6$ *m/s* (we got the latter from *Part a*). We want the pull-out acceleration a_p executed between $y_2 = 3$ *meters* and $y_3 = 1$ *meter*. Scanning the kinematic equations, we find that the same equation used in *Part a* will do the job:

$$(v_3)^2 = (v_2)^2 + 2a_p(x_3 - x_2)$$
$$\Rightarrow \quad (0)^2 = (-43.6 \text{ m/s})^2 + 2a_p[(1 \text{ m}) - (3 \text{ m})]$$
$$\Rightarrow \quad a_p = 475 \text{ m/s}^2.$$

Note 1: We used the same equation in *Parts a* and *b* even though the two situations are related to entirely different sections of motion. The moral of the story: these equations work between *ANY* two points for which you have information.

Note 2: That the acceleration is positive should not be surprising. The net force and the acceleration are proportional; it will take a positive force (i.e., a force upward in the ***+j** direction*) to stop the bats' freefall.

c.) We know the velocity at the beginning of the pull-out ($v_2 = -43.6$ *m/s*), the velocity at the end of the pull-out ($v_3 = 0$), and the acceleration through the pull-out ($a_p = 475$ *m/s*^{2}). To determine the time:

$$a = (v_3 - v_2)/\Delta t$$

$$\Rightarrow \quad \Delta t = (v_3 - v_2)/a$$
$$= [(0) - (-43.6 \text{ m/s})]/(475 \text{ m/s}^2)$$
$$= .092 \text{ seconds.}$$

4.6) $x(t) = bt^4 - ct$ (plugging in $b = 6$ and $c = 2$, we get $x(t) = 6t^4 - 2t$).

a.) The body will change its direction when reaches either its maximum or minimum *x position*. At that point it will reverse its direction. The *velocity* at the turn-around point is ZERO. Ignoring *unit vectors* (it is one dimensional motion), the *velocity function* is:

$$v = dx/dt$$
$$= d[6t^4 - 2t] / dt$$
$$= 24t^3 - 2.$$

Putting the body's *velocity at the turn-around point* equal to zero, we get:

$$24t^3 - 2 = 0$$
$$\Rightarrow \quad t = (2/24)^{1/3}$$
$$= .437 \text{ seconds.}$$

b.) If we can determine the time at which $a = +12\ m/s^2$, we can put that time back into our *velocity function* to determine the velocity of the body at that point in the motion. Knowing the *velocity* (sign and all) will tell us the *direction* of motion. Executing that plan:

$$a(t) = dv(t)/dt$$
$$= d[24t^3 - 2t]/dt$$
$$= 72t^2.$$

Putting this equal to $12\ m/s^2$ yields our time:

$$72t^2 = 12$$
$$\Rightarrow \quad t = .408 \text{ seconds.}$$

The velocity at $t = .408$ *seconds* is:

$$v = 24t^3 - 2$$
$$= 24(.408)^3 - 2$$
$$= -.37 \text{ m/s.}$$

As the *velocity vector* is negative, the body must be moving in the *negative direction* when the acceleration is $+12\ m/s^2$.

4.7) The stunt woman's velocity at *Point A* is $v_A = -25\ m/s$. The acceleration is still $a = -g$.

a.) After 2 seconds, she has moved a distance:

$$\begin{aligned}(x_B - x_A) &= v_A \Delta t + (1/2)a(\Delta t)^2 \\ &= (-25\ \text{m/s})(2\ \text{sec}) + .5(-9.8\ \text{m/s}^2)(2\ \text{sec})^2 \\ &= -69.6\ \text{meters.}\end{aligned}$$

b.) Her velocity at *Point B*:

$$\begin{aligned}v_B &= v_A + a\Delta t \\ &= (-25\ \text{m/s}) + (-9.8\ \text{m/s}^2)(2\ \text{sec}) \\ &= -44.6\ \text{m/s.}\end{aligned}$$

4.8) Because there are two cars in the system, we have two sets of kinematic equations available to us (one for the motion of each car). There are two common quantities that will link the two sets of equations during the time interval between the first pass and the second pass. They are: 1.) both cars will travel for the same amount of time during the interval; and 2.) their last-pass coordinate will be the same (we'll call this the "final" coordinate for simplicity).

If we define the origin (i.e., $x_1 = 0$) at the point where the two cars *first pass* and define the *last pass* position as x_2:

a.) For the *first car* whose initial velocity is 18 m/s and whose acceleration is zero:

$$\begin{aligned}&(x_2 - x_1) = v_{fst,1}\Delta t + (1/2)a_{fst}(\Delta t)^2 \\ \Rightarrow\quad &(x_2 - 0) = (18\ \text{m/s})t + .5(0)t^2 \\ \Rightarrow\quad &x_2 = 18t \qquad\qquad \text{(Equation 1).}\end{aligned}$$

--For the *second car* whose initial velocity is 4 m/s and whose acceleration is 6 m/s^2:

$$\begin{aligned}&(x_2 - x_1) = v_{sec,1}\Delta t + (1/2)a_{sec}(\Delta t)^2 \\ \Rightarrow\quad &(x_2 - 0) = (4\ \text{m/s})t + .5(6\text{m/s}^2)t^2\end{aligned}$$

$$\Rightarrow \qquad x_2 = 4t + 3t^2 \qquad \text{(Equation 2).}$$

Equating the two independent expressions for x_2 (i.e., Equations 1 and 2) yields:

$$18t = 4t + 3t^2.$$

Dividing by t yields:

$$18 = 4 + 3t$$
$$\Rightarrow \quad t = 4.67 \text{ seconds.}$$

b.) Using Equation 1 yields:

$$\begin{aligned} x_2 &= 18t \\ &= 18(4.67 \text{ seconds}) \\ &= 84.1 \text{ meters.} \end{aligned}$$

Using Equation 2 yields:

$$\begin{aligned} x_2 &= 4t + 3t^2 \\ &= 4(4.67 \text{ sec}) + 3(4.67 \text{ sec})^2 \\ &= 84.1 \text{ meters.} \end{aligned}$$

EITHER EQUATION WILL DO!

c.) The *second car's* velocity as it passes the first car:

$$\begin{aligned} v_{sec,2} &= v_{sec,1} + a_{sec}\Delta t \\ &= (4 \text{ m/s}) + (6 \text{ m/s}^2)(4.67 \text{ sec}) \\ &= 32.02 \text{ m/s.} \end{aligned}$$

An alternative approach would be:

$$\begin{aligned} (v_{sec,2})^2 &= (v_{sec,1})^2 + 2a_{fst}(x_p - x_o) \\ &= (4 \text{ m/s})^2 + 2(6 \text{ m/s}^2)[(84.1 \text{ m}) - 0] \\ &= 32.02 \text{ m/s.} \end{aligned}$$

EITHER APPROACH WORKS!

d.) The *second car's* average velocity:

$$v_{avg} = (v_{sec,2} + v_{sec,1})/2$$
$$= (32.02 \text{ m/s} + 4 \text{ m/s})/2$$
$$= 18.01 \text{ m/s}.$$

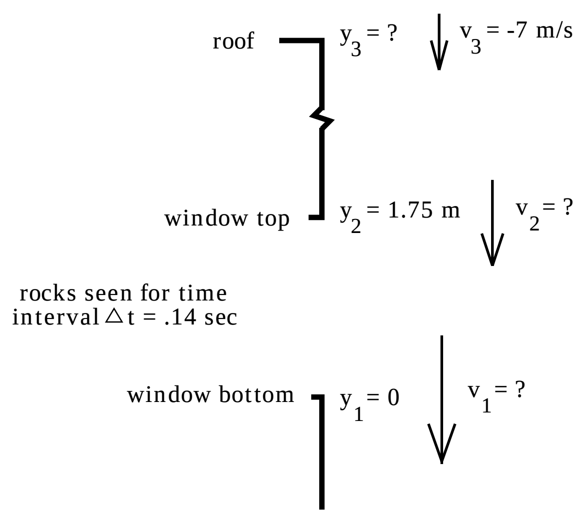
e.) Time for the *second car* to travel to a velocity of 100 m/s:

$$a_{sec} = (v_{100} - v_{sec,1})/\Delta t$$
$$\Rightarrow \quad Dt = (100 \text{ m/s} - 4 \text{ m/s})/(6 \text{ m/s}^2)$$
$$= 16 \text{ seconds}.$$

4.9)

a.) All the known information has been inserted into the sketch to the right. The coordinate axis has been placed so that $y_1 = 0$ at the bottom of the window. We will proceed by writing out kinematic relationships that will allow us to solve for information we need.

To determine the velocity of the rock when it is at the *top of the window*, we will define the "final" position to be at y_1 with an "initial" position at y_2. With that, we can write:



$$(y_1 - y_2) = v_2 \Delta t + (1/2)a(\Delta t)^2$$
$$(0) - (1.75 \text{ m}) = v_2(.14 \text{ s}) + (1/2)(-9.8 \text{ m/s}^2)(.14 \text{ s})^2$$
$$\Rightarrow \quad v_2 = [(-1.75 \text{ m}) - .5(-9.8\text{m/s}^2)(.14 \text{ s})^2]/ (.14 \text{ s})$$
$$= -11.8 \text{ m/s}.$$

Note: The negative sign makes sense considering the fact that the rock is moving *downward*.

b.) We want the distance between the *top of the building* and the *bottom of the window* (this will numerically equal y_3). An equation with known values and y_3 is (note that the "final" position here is y_2):

$$(v_2)^2 = (v_3)^2 + 2 \quad a \quad (y_2 - y_3)$$
$$(-11.8 \text{ m/s})^2 = (-7 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)[(1.75 \text{ m}) - y_3)]$$
$$\Rightarrow \quad y_3 = 6.35 \text{ meters.}$$

c.) A change in the initial velocity of the rock does nothing to the acceleration of the rock. Once it becomes free, the rock picks up velocity due to gravity at a rate of *9.8 m/s* <u>every second</u>, no matter what. Tricky, eh?

4.10) The car's initial velocity is $v_1 = 40$ *m/s*; its pedal-to-the-metal acceleration is $a_{go} = +3$ *m/s*^{2} and its stopping acceleration is $a_{stop} = -3$ *m/s*^{2}. The light stays yellow for 1.2 seconds before turning. We need to consider two situations: a.) what is the maximum distance the car can be from the restraining line if it is to successfully accelerate all the way through the intersection in 1.2 seconds; and b.) what is the minimum distance the car can be from the restraining line if it is to brake successfully?

a.) Pedal-to-the-metal: Taking Δx_{go} to be the distance from the restraining line to the point at which the car begins its acceleration, the car must go Δx_{go} *+ 18 meters* to make it *through the intersection* without incident (remember, the intersection extends 18 meters beyond the restraining line). With 1.2 seconds to accomplish the feat, we can write:

$$(\Delta x_{go} + 18) = v_1 \Delta t + (1/2)a_{go}(\Delta t)^2$$
$$\Rightarrow \quad \Delta x_{go} = (-18 \text{ m}) + (40 \text{ m/s})(1.2 \text{ sec}) + .5(+3 \text{ m/s}^2)(1.2 \text{ sec})^2$$
$$= 32.16 \text{ meters.}$$

If the car is closer than 32.16 meters, it can accelerate and still make it across the intersection. If the car is farther than 32.16 meters, it will not be able to accelerate completely through the intersection.

b.) Braking: The 1.2 seconds is useless in this section. It doesn't matter whether the car is sliding while the light is red or not. All that matters is that the car stop just behind the restraining line. Putting the origin at the restraining line (i.e., $x_{r.l.} = 0$), and defining x_{brake} to be the position at which the brakes must be hit to effect the stop right at the restraining line, we can write:

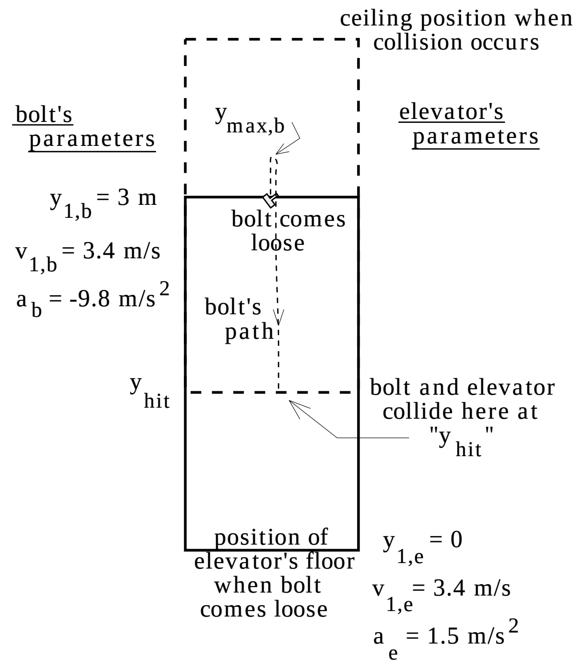
$$v_{r.l.}^2 = v_{brk}^2 + 2a(x_{r.l.} - x_{brk})$$
$$\Rightarrow \quad x_{brk} = -(v_{r.l.}^2 - v_{brk}^2) / (2a)$$
$$= -[0^2 - (40\text{ m/s})^2] / [2(-3\text{ m/s}^2)]$$
$$= -267\text{ meters}.$$

Note: The negative sign simply means that a car moving to the right in the positive direction must begin to stop to the *left* of the origin (the restraining line).

c.) Bottom Line: The car eats it no matter what if it is between 267 meters and 32 meters of the restraining line.

4.11)

a.) We need a *fixed* coordinate axis from which to make our measurements. We could take ground level as our $y = 0$ level, but instead we will take the position in space of the *elevator's floor* just as the bolt releases (this will be useful because it is the elevator's floor that will come in contact with the bolt at the *collide* position).



The bolt's initial velocity is the same as that of the elevator at the moment it becomes free. That velocity is upward, so the bolt will move above its *suddenly-free* position to some maximum position y_{max} before beginning to descend. WHENEVER YOU ARE LOOKING FOR y_{max}, ALWAYS USE:

$$v_{top}^2 = v_1^2 + 2a(y_{top} - y_1).$$

As we know the bolt's velocity at the top of its motion is $v_{top} = 0$, we can write:

$$v_{top}^2 = v_{1,b}^2 + 2a_b(y_{top} - y_{1,b}).$$
$$\Rightarrow \quad (0)^2 = (3.4\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)[y_{top} - (3\text{ m})]$$
$$\Rightarrow \quad y_{top} = 3.59\text{ meters}$$
$$\Rightarrow \quad y_{max\ above\ ground} = y_{top} + 4 = 7.59\text{ meters}.$$

b. and c.) Finding the *time of flight* Δt and the *final coordinate position* y_{hit} requires two equations solved simultaneously. As such, we will do both *Parts b* and *c* in this section.

The first thing to notice is that this is really two problems happening at the same time. During the bolt's freefall, the elevator is accelerating upward by its motor while the bolt is accelerated downward by gravity. Treating the two entities as individuals:

i.) For the elevator's motion (or, at least, the motion of the elevator's floor), assuming $y = 0$ is the floor's position when the bolt releases:

$$\begin{aligned} y_{hit} &= y_{1,e} + v_{1,e}\Delta t + (1/2)\ a_e\ (\Delta t)^2 \\ &= (0) + (3.4\text{ m/s})t + .5\ (1.5\text{ m/s}^2)\ t^2 \qquad \text{(Equ. A).} \end{aligned}$$

ii.) For the bolt's motion, assuming that we are using the same coordinate axis as above so that the bolt's initial position is $y = 3$ *meters*:

$$\begin{aligned} y_{hit} &= y_{1,b} + v_{1,b}\Delta t + (1/2)\ a_b\ (\Delta t)^2 \\ &= (3\text{ m}) + (3.4\text{ m/s})t + .5\ (-9.8\text{ m/s}^2)\ t^2 \qquad \text{(Equ. B).} \end{aligned}$$

iii.) Equating *Equations A* and *B* (y_{hit} is the same in both equations):

$$\begin{aligned} (3.4\text{ m/s})t + .5(1.5\text{ m/s}^2)\ t^2 &= 3 + (3.4\text{ m/s})t + .5(-9.8\text{ m/s}^2)t^2 \\ \Rightarrow\quad 5.65t^2 &= 3 \\ \Rightarrow\quad t &= .73\text{ seconds.} \end{aligned}$$

iv.) Using *Equation A* to determine y_{hit}:

$$\begin{aligned} y_{hit} &= y_{1,e} + v_{1,e}\ \Delta t + (1/2)\ a_e\ (\Delta t)^2 \\ &= (0) + (3.4\text{ m/s})(.73\text{ s}) + .5\ (1.5\text{ m/s}^2)\ (.73\text{ s})^2 \\ &= 2.88\text{ meters.} \end{aligned}$$

v.) The bolt was initially at $y = 3$ *meters*. When it hits the floor, its coordinate is $y_{hit} = 2.88$ *meters*. The distance the bolt actually falls is:

$$\begin{aligned} \Delta y &= y_{final} - y_{initial} \\ &= (2.88\text{ m}) - (3.0\text{ m}) \\ &= -.12\text{ meters.} \end{aligned}$$

vi.) For the amusement of it, let's check our y_{hit} value by solving *Equation B*:

$$y_{hit} = y_{1,b} + v_{1,b} \Delta t + (1/2) a_b (\Delta t)^2$$
$$= (3\text{ m}) + (3.4\text{ m/s})(.73\text{ s}) + .5(-9.8\text{ m/s}^2)(.73\text{ s})^2$$
$$= 2.87\text{ meters.}$$

. . . close enough for government work.

d.) The magnitude of the velocity of the bolt, relative to a fixed frame of reference (i.e., *not* relative to the elevator's floor) is determined as follows:

$$v_{b,bot}^2 = v_{1,b}^2 + 2 a_b [y_{b,bot} - y_{1,b}]$$
$$= (3.4\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)[(2.87\text{ m}) - (3\text{ m})]$$
$$\Rightarrow \quad v_{b,bot} = 3.76\text{ m/s.}$$

As a vector, this would be $\mathbf{v}_{b,bot} = -(3.76\text{ m/s})\mathbf{j}$.

4.12)

a.) In projectile motion, questions like, "How far (Δx)" and "How long (Δt)" are usually approached simultaneously. In this case, the *time* variable is most easily determined by looking at the motion in the *y direction* only. Noting that the ball started at $y_1 = 1.3$ *meters*, ended at $y_2 = 0$ (i.e., at ground level), and had a *y component* velocity of $v_y = 41 \sin 50^o$, we can write:

$$(y_2 - y_1) = v_{1,y}t + (1/2)a_y t^2$$
$$(0\text{ m} - 1.3\text{ m}) = (41 \sin 50^o)t + .5(-9.8\text{ m/s}^2)t^2$$
$$-1.3 = 31.4t - 4.9t^2$$
$$\Rightarrow \quad 4.9t^2 - 31.4t - 1.3 = 0.$$

Using the Quadratic Formula, we get:

$$t = [-b \pm [b^2 - 4ac]^{1/2}]/2a$$
$$= [-(-31.4) \pm [(-31.4)^2 - 4(4.9)(-1.3)]^{1/2}]/[2(4.9)]$$
$$= [31.4 \pm (31.8)]/(9.8)$$
$$= 6.45\text{ seconds.}$$

b.) Assuming $x_1 = 0$ is the *x coordinate* at which the ball is struck, the net horizontal distance traveled will be:

$$(x_2 - x_1) = v_{1,x}t + (1/2)a_x t^2$$
$$(x_2 - 0) = (v_1 \cos q)t + .5(0 \text{ m/s}^2)t^2$$
$$\Rightarrow \quad x_2 = (41 \text{ m/s})(\cos 50^o)(6.45 \text{ sec}) + 0$$
$$= 170 \text{ meters.}$$

c.) Height is a *y* related quantity. Knowing that the *y component* of velocity at the top of the arc (i.e., at y_{max}) will be zero, we can write:

$$(v_{max,y})^2 = (v_{1,y})^2 + 2a_y(y_{max} - y_1)$$
$$\Rightarrow \quad y_{max} = [(v_{max,y})^2 - (v_{1,y})^2 + 2a_y(y_1)]/2a_y$$
$$= [(0)^2 - (41 \sin 50^o)^2 + 2(-9.8 \text{ m/s}^2)(1.3 \text{ m})]/[(2)(-9.8 \text{ m/s}^2)]$$
$$= 51.6 \text{ meters.}$$

d.) At the end of the flight, the ball has an *x component* of velocity that has not changed throughout the motion (we are neglecting air friction and there are no other natural forces out there to make the *x motion* change). That value will be:

$$v_{2,x} = (41 \text{ m/s})(\cos 50^o)$$
$$= 26.4 \text{ m/s.}$$

The *y motion* velocity has changed because gravity has been accelerating the ball throughout the motion. We can get the *y* velocity using:

$$(v_{2,y})^2 = (v_{1,y})^2 + 2a_y(y_2 - y_1)$$
$$\Rightarrow \quad v_{2,y} = [(v_{1,y})^2 + 2a_y(y_2 - y_1)]^{1/2}$$
$$= \left[[(41 \text{ m/s})(\sin 50^o)]^2 + 2(-9.8 \text{ m/s}^2)(0 - 1.3 \text{ m})\right]^{1/2}$$
$$= 31.8 \text{ m/s.}$$

Note: This equation yields <u>magnitudes only</u>. The *y component* of the "final" velocity is in the ***-j** direction*. That means $v_{2,y} = -31.8 \text{ m/s}$.

Putting it all together:

$$\mathbf{v}_2 = (26.4\mathbf{i} - 31.8\mathbf{j}) \text{ m/s.}$$